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1989 J. Phys. A: Math. Gen. 22 4659

(http://iopscience.iop.org/0305-4470/22/21/026)

J. Phys. A: Math. Gen. 22 (1989) 4659-4664. Printed in the UK

Dynamical phase transition of spin glasses in a magnetic field

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Received 31 May 1989

Abstract. We study the time evolution of the damage and the macroscopic distance between two configurations of three-dimensional $\pm J$ Ising spin glasses in an external magnetic field. Similarly to the zero-field case, a phase transition and three temperature regimes are found for the damage. The multifractal properties of the distribution of probability for a site to be damaged n times are also investigated at the onset of the chaotic phase.

In recent years remarkable progress has been made in establishing a mean-field theory for spin glasses [1]. The spin-glass phase has been found to be characterised by a multitude of pure thermodynamic ground states, not simply related by symmetry and with equal order parameter associated to each state. Moreover, a transition has been found to happen in a finite magnetic field along the so called de Almeida–Thouless (AT) line [2], which separates the region of the phase diagram with many states (spin-glass phase) from the region with a single state (paramagnetic phase).

Unfortunately, in the finite-dimensional case the situation has not yet reached a clear standpoint. It is now quite well established that the lower critical dimension d_l for Ising spin glasses in between 2 and 3 (for a recent review see [3]), and the spin-glass phase has been demonstrated [4, 5] in $d=3$. On the other hand, the nature of this phase is still a debated question; that is, whether the phase space can be pictured with many valleys separated by infinitely high barriers, like in mean-field theory, or whether it has a simpler structure with only two thermodynamic states [6] related by spin-flip symmetry. Moreover, the existence of a phase transition in a non-zero magnetic field is also controversial. If the mean-field picture for the spin-glass phase is valid, then the analogue of the AT line should be found also in finite dimensions and a phase transition would occur along a critical line $T=T_c(H)$. If instead there are only two thermodynamic states at $H=0$, then no transition should occur in a finite field for sufficiently long observation times.

Recently, methods developed in the study of dynamical systems, such as cellular automata [7], have been applied to spin systems. The idea is to consider the time evolution of two parallel configurations A and B of the same system of N spins and to evaluate the damage or Hamming distance

$$\langle D(t)\rangle = \frac{1}{4N}\sum_{i=1}^{N}\langle (s_i^{\mathrm{A}}(t)-s_i^{\mathrm{B}}(t))^2\rangle \qquad (1)$$

where $s_i^{\mathrm{A}}(t)$ and $s_i^{\mathrm{B}}(t)$ are the values of the spin on site i in configurations A and B

‖ Laboratoire de l'Institut de Recherche Fondamentale du Commissariat à l'Energie Atomique.

0305-4470/89/214659+06$02.50

at time t and the brackets $\langle \ldots \rangle$ denote the time average over the interval $]t-\Delta t, t]$. This distance $\langle D(t)\rangle$ depends on the initial value $D(0)$ and it represents how the two configurations evolve in phase space. Starting with an infinitesimally small $D(0)$, $\langle D(t)\rangle$ will go asymptotically to zero in the so-called frozen phase, whereas it will tend to a value different from zero in the chaotic phase.

These ideas were first applied to the two-dimensional Ising model using Glauber dynamics [8] and a transition from a frozen to a chaotic phase was detected at the value of the Curie temperature. In this case the two configurations A and B were submitted to the same thermal noise; that is, the same random number was used to determine the state of spin i in the two configurations.

For the case of the heat bath dynamics, where the probability for the spin i to be up is

$$p_i(t) = 1 - \exp\left(-\frac{2}{kT}\sum_j J_{ij}s_j(t)\right)$$

the sum being over the nearest neighbours of site i, exact results were derived [9] if a spin was kept up at the origin of one configuration at all times. It was in fact found that the spin-spin correlation function at a distance r from the origin is related to the probability for a site at that distance to become damaged. If instead the boundaries of configurations A and B were fixed up and down respectively, the damage $\langle D(t)\rangle$ was found to be related to the magnetisation.

The concept of damage was also considered for symmetric [10, 11] and non-symmetric [11, 12] spin glasses. For the three-dimensional symmetric $\pm J$ Ising spin glasses and heat bath dynamics, the study of the damage $\langle D(t)\rangle$ for short observation times ($t = 500$) has detected [10] three different regimes: for temperatures $T > T_1 \simeq 4.1$, $\langle D(t)\rangle$ vanishes independently of the initial damage $D(0)$; for $T_1 > T > T_2 \simeq 1.8$, $\langle D(t)\rangle$ is different from zero and independent of $D(0)$; for $T < T_2$, $\langle D(t)\rangle$ is different from zero but depends on the initial damage.

In attempting to investigate the phase space properties of spin glasses, a recent paper [13] has studied, together with the damage $\langle D(t)\rangle$ which represents the microscopic distance between the two configurations, the macroscopic distance between them

$$d_{AB}(t) = \frac{1}{4N}\sum_i \langle s_i^A(t) - s_i^B(t)\rangle^2 \qquad (2)$$

where the sum over i implies here and in equation (1) an average over the bond configurations. This distance d_{AB} was found to go to zero at a temperature close to the known value for the spin-glass transition ($T_{sg} \simeq 1.2$) and to be sensitive to the initial conditions below this temperature.

These results have been interpreted as indications that a mean-field-like multivalley structure holds also for three-dimensional Ising spin glasses. Moreover, the distribution of probabilities for a site to go from a healed configuration to a damaged one has been introduced. This distribution has multifractal scaling properties (see [14] for a recent review) for the 3D $\pm J$ Ising spin glasses at the pure Ising critical temperature T_c whereas it is characterised by simple constant gap scaling at $T = T_{sg}$, in the pure ferromagnetic Ising model at T_c and in the 2D Ising spin glass.

In this paper we want to extend these concepts to the study of the three-dimensional $\pm J$ Ising spin glass in a non-zero homogeneous magnetic field to investigate whether a transition occurs and what are the scaling properties of the damage at the onset of the chaotic phase.

In order to do so, we perform numerical simulations of a 3D Ising spin glass on a cubic lattice of linear size L with periodic boundary conditions. The interactions $J = \pm 1$ are assigned initially at random and quenched in time. At temperature T and at a value $H = 1$ of the magnetic field, the probability for the spin i to be up at time t is given by the heat bath dynamics

$$p_i(t) = \left\{1 + \exp\left[-\frac{2}{kT}\left(\sum_j J_{ij} s_j(t) + H s_i(t)\right)\right]\right\}^{-1}. \tag{3}$$

After thermalisation of typically several thousand steps of an initial random configuration A of spins, we construct its parallel image B where all the spins are identical except for a fraction $D(0)$ which is reversed. The two configurations A and B will then evolve in time according to equation (3), with the same random number determining the state of spin i in A and B. The calculated quantities are first averaged over the time interval with $\Delta t = 500$ time steps, then averaged over several configurations of quenched disorder.

For each configuration we look at $q = q_{AA} = q_{BB} = \Sigma_i \langle s_i(t)\rangle^2$, which for $H = 0$ is the Edwards-Anderson (EA) order parameter. Due to the orientation of the spins in the field, we do not expect q to be an order parameter for a spin-glass phase in the field. This quantity is found to be independent of the particular configuration and of the initial damage chosen.

Moreover, we calculate the damage $\langle D(t)\rangle$ and the macroscopic distance d_{AB} (figure 1). As in the zero-field case, three regions can be detected for the damage which goes to zero at $T_1' \simeq 4.3$ and becomes independent of the initial damage at $T_2' \simeq 2.1$. This

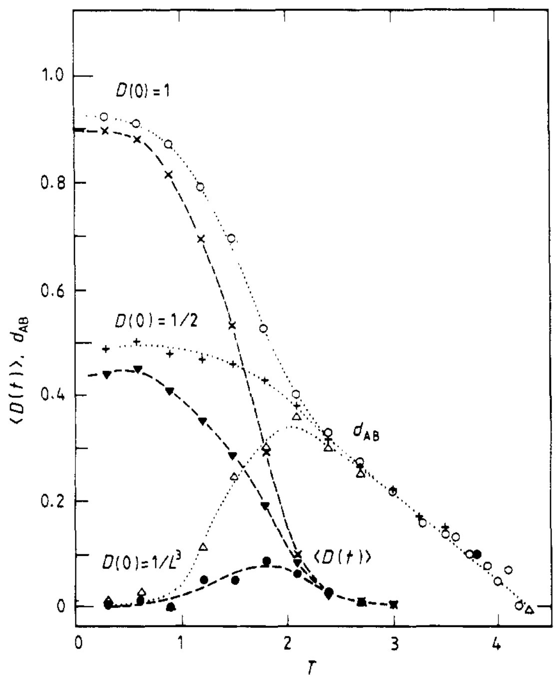

Figure 1. The damage $\langle D(t)\rangle$ (broken curves) and the macroscopic distance d_{AB} (dotted curves) as functions of the temperature T for 150 configurations of the system size $L = 10$ in a field $H = 1.0$ after 1000 time steps of observation and 5000 time steps of thermalisation for different initial damages $D(0)$. The transition occurs at $T \simeq 2.1$.

value is larger than the transition value $T_2 \simeq 1.8$ found in the zero-field case [10, 13], with comparable numerical effort. The macroscopic distance d_{AB} is instead found to go to zero close to T_2' and it depends on the initial choice for $D(0)$ below this temperature. As opposed to q, the quantity d_{AB} retains the same meaning here as in the zero-field case since the eventual field-induced magnetisation is cancelled in equation (2). The sensitivity to the initial conditions can be interpreted, as in the zero-field case, as an indication that a multivalley structure of the spin-glass phase holds also in a finite field. Moreover, the evidence for two critical temperatures, T_1' and T_2', would support the opinion that the analogue of an AT line might exist in the finite-dimentional case, separating the region of phase space with a single valley from the region with a more complicated structure.

In analogy with the mean-field case like the SK model [3], one could have expected the transition value to become smaller in the presence of the field. On the other hand, in some simplified mean-field models transition lines have been found where the critical temperature increases with the field [15], or even where T_c increases for some values of the field and then decreases [16]. In the present case, the observation that the transition temperature is larger than in the zero field could also be due to the relaxation times being longer in the finite field than in the zero field.

To this extent, we have checked the stability of the quantities q_{AA}, q_{BB}, d_{AB} and $\langle D(t)\rangle$ with time. Figure 2 shows these quantities for one configuration of the system size $L = 30$ with an initial damage $D(0) = \frac{1}{2}$ as function of time. Contrary to the zero-field case, where the data decrease very slowly but systematically with time, here no sensible decay is detected over the same observation time.

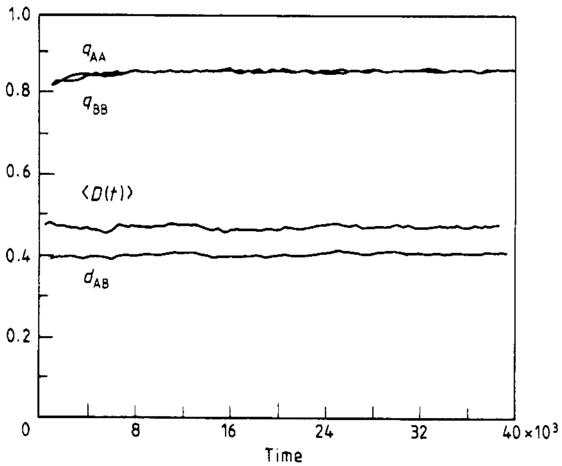

Figure 2. The EA order parameters q_{AA} and q_{BB}, the Hamming distance $\langle D(t)\rangle$ and the macroscopic distance d_{AB} as functions of time for one configuration of the system size $L = 30$ with an initial damage $D(0) = 1/2$, at temperature $T = 0.9$ and in a magnetic field $H = 1.0$.

We wish, however, to stress that full thermalisation has not been achieved and that we have not analysed the dependence of the various quantities on the time interval Δt. The above interpretation should be therefore considered as a working scheme which needs more careful analysis [17] to be fully tested.

Finally, at the onset of the chaotic phase $T_1' \simeq 4.3$, we have studied the distribution of probability for a site i to be damaged. After fixing the central site in configuration A to be up, i.e. $s_0^A = +1$ at all times, we calculate for each site i the number of times this site changes from a healed configuration to a damaged one, n_i. The probability to become damaged is then given by $p_i = n_i / \Sigma_i n_i$. We evaluate the distribution of these probabilities and its moments

$$M(q) = \sum_i p_i^q \sim L^{-\tau(q)}. \tag{4}$$

These moments are found to have multifractal [14] scaling properties (figure 3); that is, the exponents $\tau(q)$ are not a simple linear function of q as for constant-gap scaling, but have a more complicated dependence. This behaviour is an expression of the fact that the distribution is not strongly peaked about its most probable value but spreads over a wide range of probabilities. Then the subsets of sites which are damaged with very high probability have their own scaling, independently of the behaviour of the almost-frozen sites. However, we notice that the deviation of the values of the critical

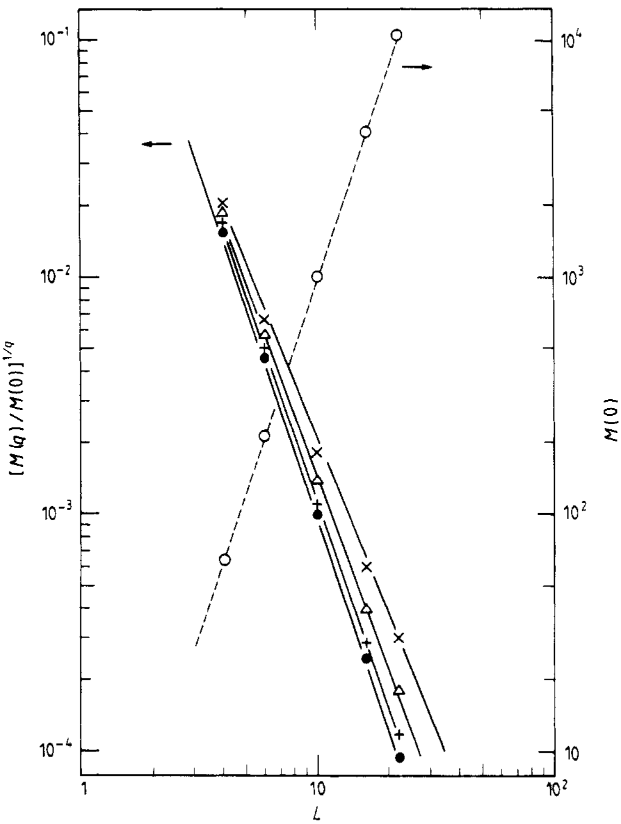

Figure 3. The moments of the distribution of probability of getting damaged plotted on a log-log scale as the renormalised quantities $[M(q)/M(0)]^{1/q}$ against L for a magnetic field $H = 1.0$ and at temperature $T = 4.3$, for $q = 0$ (○), 1 (●), 2 (+), 3 (△) and 4 (×). The data are taken after 2000 time steps of thermalisation and 25 000 steps of observation time. The average is over 1000, 600, 300, 120 and 50 samples for the system sizes $L = 4$, 6, 10, 16 and 22 respectively. Since the data for different q do not fall on parallel straight lines, constant gap scaling is to be excluded.

exponents from a linear dependence on q is less pronounced in the present case than in the zero-field case [13].

In conclusion, we have extended the concepts of spreading the damage and of macroscopic distance between states d_{AB} to the three-dimensional spin glasses in a non-zero magnetic field. Similarly to the zero-field case, a transition and three regimes are detected for the damage. Our data for rather limited observation times give an indication that the analogue of the AT line would also occur in finite dimensions. Moreover, as in the zero-field case, the probability distribution of becoming damaged is multifractal at the onset of the chaotic phase. It would be interesting to further investigate the stability of the low-temperature phase with respect to the external field and for longer observation times.

Acknowledgments

We would like to thank J de Almeida and I Kondor for urging us to investigate the damage-spreading problem in the finite-field case. We also acknowledge stimulating discussions with G Parisi and N Sourlas.

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